

# MAE106 Homework 2

## DC Motors & Intro to the frequency domain

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### Problem 1

You are given the circuit shown in Figure 1. The battery outputs 6V and the potentiometer is rated at 100k $\Omega$ . Assume that the motor is not loaded and friction in the motor is negligible. Also assume that the battery and potentiometer can handle any current that the circuit needs. Show your work for all the questions below.

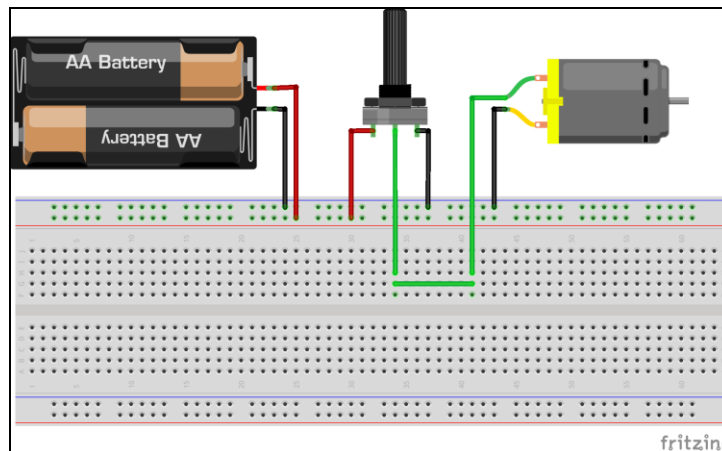


Figure 1. Circuit schematic for problem 1

1. You turn the potentiometer so that the resistance across the wiper and ground is set at 25k $\Omega$ . In this settings the spins at 120 revolutions per minute (rpm) and has an effective resistance of 5k $\Omega$ , what is the voltage across the motor?
2. You now turn the potentiometer and measure the voltage across the motor to be 5.3V. What is the resistance across the potentiometer's wiper and ground leads? What is the speed, in rpm, at which the motor is spinning?
3. Draw the motor's velocity as a function of its input voltage. Label the equation for the line that you draw. For this circuit, what is the maximum speed at which the motor can spin?

## Problem 2

In class we studied the mathematical model of a DC brushed motor and described it as:

$$V = L \frac{di}{dt} + Ri + B\dot{\theta}$$

we then derived the torque vs. time equation for a motor that was stalled and found it to be:

$$Torque = \frac{BV}{R} (1 - e^{-\frac{t}{\tau}})$$

You have been tasked with finding a 12V DC brushed motor that can attain a steady state stall torque of  $2.4N \cdot m$ . This motor must also be able to reach 95% of its steady state stall torque in  $20msec$  or less.

Show all your work for the questions below.

1. You find a supplier whose motors are all rated at a resistance of  $100\Omega$ . What is the minimum value of the motor's torque constant that satisfies the required stall torque?
2. What is the maximum value of the motor's inductance that satisfies the required response time of  $20msec$ ? (hint: the motor needs  $(3 \cdot \tau)$  seconds to reach 95% of its steady state value)
3. What is the no-load speed of this motor, in rpm, at 12V? Draw the torque vs. speed curve for this motor when it runs at 12V. Label the equation for the line that you draw.

### Problem 3

During this week's lab you used an RC circuit as a low-pass filter in order to filter the PWM signal generated with the microcontroller. The circuit is shown below:

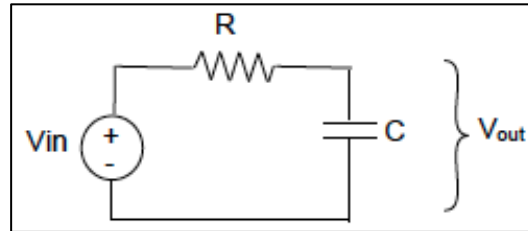


Figure 2. Circuit for problem 3

Show all your work when answering the following questions.

1. Use Kirchhoff's voltage law to find the first-order differential equation that relates the voltage across the capacitor ( $V_{out}$ ) to  $V_{in}$ ,  $R$ , and  $C$ .
2. Using the first-order differential equation above, find the transfer function of the system. Assume all initial conditions are zero. (hint: you will need to use the Laplace transform to go from the time domain to the frequency domain).
3. Use the transfer function derived above to find the response of the system, in the time domain, to a step change in voltage from 0 to 5V (Let  $R = 15\ \Omega$  and  $C = 0.8\text{F}$ ). Draw the voltage vs. time curve for this step change and label the time constant and the value of the voltage at this time constant. (hint: you will need to use the inverse Laplace transform to go from the frequency domain back to the time domain; also remember that  $L\{\text{Unit step } 1(t)\} = \frac{1}{s}$ ).

## Resources

<http://lancet.mit.edu/motors/motors3.html>

<http://www.micromo.com/technical-library/dc-motor-tutorials/motor-calculations>

[http://www.gearseds.com/files/Lesson3\\_Mathematical%20Models%20of%20Motors.pdf](http://www.gearseds.com/files/Lesson3_Mathematical%20Models%20of%20Motors.pdf)

[http://www.rapidtables.com/convert/electric/Volt\\_to\\_Watt.htm](http://www.rapidtables.com/convert/electric/Volt_to_Watt.htm)