MAE106 Homework 3
Low-pass filters and block diagrams

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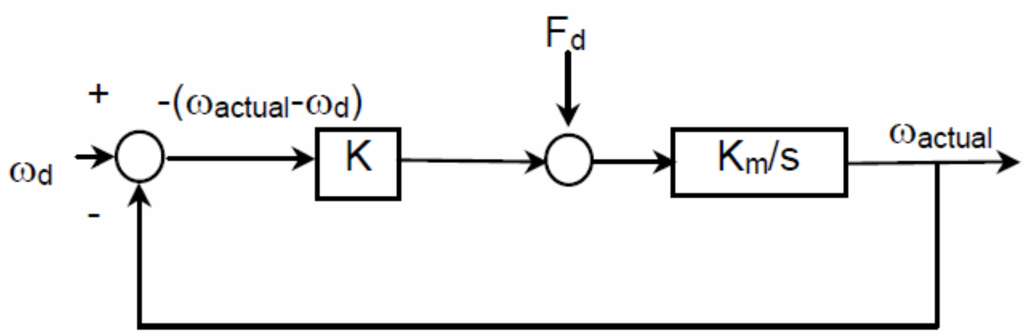
For problems 1 and 2 use the following block diagram.



Problem 1

Derive an expression that relates ω_{actual} to ω_d and F_d. Explain why high-gain feedback (i.e. a big proportional gain K) was able to better cancel the effects of friction on the motor shaft and make the motor spin at a speed closer to the desired velocity?

Solution

The control law relating K and ω is given by:

$$F = K(\omega_d - \omega)$$

adding the disturbance force we then have:

$$\omega = (K(\omega_d - \omega) + F_d)\frac{K_m}{s}$$

$$\frac{KK_m}{s}\omega_d + \frac{K_m}{s}F_d = \omega + \frac{KK_m}{s}\omega = \omega\left(\frac{s + KK_m}{s}\right)$$

$$\omega = \frac{KK_m}{s + KK_m}\omega_d + \frac{K_m}{s + KK_m}F_d$$

therefore if K is large, then the effect of the disturbance force is effectively cancelled and $\omega \sim \omega_d$

Problem 2

Derive the transfer function of the closed-loop system shown in the block diagram. What type of filter does this transfer function resemble? What is its time constant? Recall that transfer functions are found by setting all inputs and disturbances to zero.

Solution

Setting the disturbance to zero we have that:

$$\omega = \frac{K_m}{s} K(\omega_d - \omega)$$

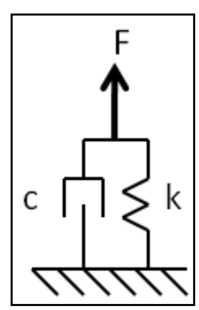
$$\frac{\omega}{\omega_d} = \frac{K_m K}{s + K_m K} = \frac{1}{\frac{1}{K_m K} s + 1}$$

The time constant of the system is given by: $\tau = 1/K_m K$

The transfer function is similar to that of a low-pass filter.

Problem 3

You are part of a team that is developing an autonomous car for a competition. A big part of the autonomy of the car comes from using algorithms from data gathered by cameras placed at the top of the car. Your task is to design a base for the cameras that will prevent them from shaking as the car drives around (*figure on the right:* the spring and damper represent the base of the camera and the ground represents the roof of the car).



For a portion of the competition the car will have to drive through gravel at 50mph. You characterize the shaking that the cameras will experience by measuring the vibration at the top of the car using an accelerometer.

The accelerometer readings reveal that the top of the car vibrates at frequencies between 45 and 60Hz.

Assume that the vibrations are purely in the vertical direction (as shown in the figure above) and that the stiffness of the base is given as: 500 N/m.

1. What is the value for C, in N.sec/meter, that will give you a cutoff frequency of 35Hz.
2. Draw the frequency response plot (Gain vs. frequency plot) for this system by computing the gain for the following frequency values: 0Hz, 5Hz, 10Hz, 20Hz, 35Hz, 50Hz, 100Hz, 500Hz, 1000Hz.
3. Again, draw the frequency response plot, but this time use the log(frequency) for the x-axis. Why is it useful to use the logarithmic scale for frequencies?