MAE106 Homework 4
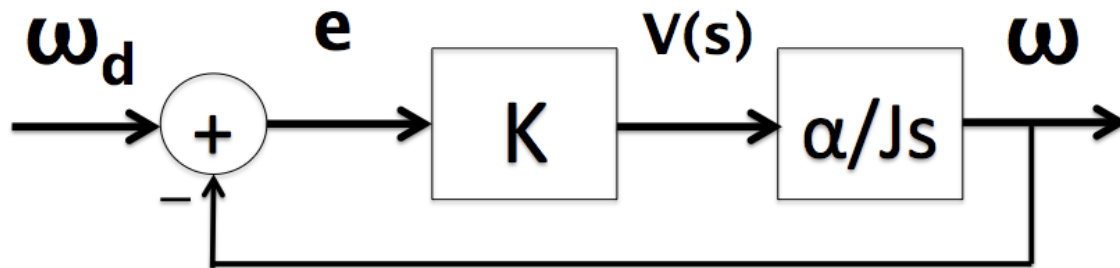# Frequency response plots and PI controller

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Problem 1

In class we looked at the block diagram for a DC motor coupled to a P-type controller (shown here). Answer the following questions based on this block diagram:

1. What is the control law in this block diagram?
2. What is the transfer function relating ω and ω_d?
3. What is the time constant for this system? How will the time constant change if we double K while leaving all other factors constant?

Solution

1. Control law:

$$V = Ke = K(\omega_d - \omega) = -K(\omega - \omega_d)$$

2. Transfer function:

$$(\omega_d - \omega) * (K) * \left(\frac{\alpha}{Js}\right) = \omega$$

$$\omega_d \left(\frac{K\alpha}{Js}\right) = \omega + \left(\frac{K\alpha}{Js}\right)\omega$$

$$\frac{\omega}{\omega_d} = \frac{K\alpha}{Js + K\alpha} = \frac{1}{\frac{J}{K\alpha}s + 1}$$

3. The time constant is the term in front of the s-term:

$$\tau = J/K\alpha$$

if we double the value of K then the time constant will be half of the previous time constant.

Problem 2

We also looked at how to create the frequency response plots for a first order system. Assume you have the following transfer function:

$$G(s) = \frac{1}{\tau s + 1}$$

Let $\tau = 1, 2, and\ 4$.

1. Plot the amplitude ratio (M) vs. frequency (ω) for frequency values of $\omega = 0, 0.5, 1, 2, 4, 8, 16, 32$. Your plot should contain three lines, one for each value of τ. Make sure to label each line accordingly.
    a. Which value of τ has the largest cutoff frequency?
2. Plot the phase angle (φ, in degrees) vs. frequency (ω) for frequency values of $\omega = 0, 0.5, 1, 2, 4, 8, 16, 32$.
3. Re-create the plot in part 1 but this time transform the y-axis to decibels () and the x-axis using the log scale.
4. Re-create the plot in part 2 but this time transform the x-axis using the log scale.

Problem 3

We also looked at the block diagram for a DC motor when we added a disturbance, such as friction (shown below).
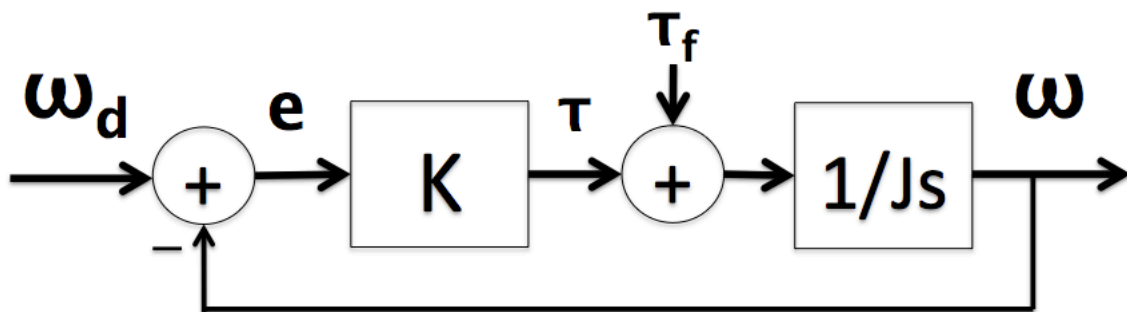
1. What is the control law in this block diagram?

2. Using the block diagram, derive the steady-state error in velocity due to friction. (hint: remember that $J\dot\omega = \tau - \tau_f$).
3. How can you change the control law from part 1 to get rid of this steady state error? Write down the equation.

Solution

1. Control law:

$$\tau = Ke = K(\omega_d - \omega) = -K(\omega - \omega_d)$$

2. Steady-state error

$$J\dot\omega = \tau - \tau_f = Ke - \tau_f$$

because we are in steady-state we know that $\dot\omega = 0$ and therefore

$$\tau_f = Ke \Rightarrow e = \frac{\tau_f}{K}$$

note that depending on how we define e we could flip the sign on the error.

3. To get rid of the steady-state error we can add an integral controller. The control law is then of the form:

$$\tau = Ke + K_i \int e\, dt$$