

MAE106 Laboratory Exercises

Lab # 6 - Vibrating systems

Goals

Understand how the oscillations in a mechanical system affect its behavior.

Parts & equipment

Qty	Part/Equipment
1	Seeeduino board
1	Motor driver
1	DC motor with encoder
1	Potentiometer
1	Accelerometer
Various	Rubber bands
1	Vibration unit

Introduction

Why study vibrations? Vibration is a term engineers use to describe oscillations in a system. Understanding vibrations is important for a wide range of engineering design. In some applications, such as car, bridge, building, and machine design, you often want to design the system so that it vibrates as little as possible. In other applications, you might want to design a system that uses vibrations to advantage, such as for radios, musical instruments, and medical equipment such as magnetic resonance imaging devices.

How we will study vibrations? In this laboratory exercise you will look at the vibrations of a cantilevered beam that supports a motor with an unbalanced load attached at its end. This system represents many typical problems in vibrations. For instance, a large rotating machine attached to a building floor can be analyzed in the same manner as this experiment, with the floor taking the place of the cantilevered beam. Alternatively, a building shaking during an earthquake can also be represented by the same equations, with the building acting as the beam itself.

Second order systems and vibrations: This lab deals with a second order system. In other words, the differential equation that describes the system has second derivatives, and the transfer function has s^2 terms. Many mechanical systems behave as second order systems because of Newton's second law ($F = ma$) in which acceleration is the second derivative of position. In fact, you can view a vast number of mechanical and control systems as second order linear mass, spring, damper systems. Thus, developing intuition about how second-order systems behave is very important. One key difference between second and first order systems, as you will see in this lab, is that second order systems can oscillate (i.e. vibrate). First order systems cannot oscillate.

Part I: Time domain analysis of the beam (Transient response)

In this part of the lab, you will measure how the beam responds to an impulsive input. This is known as the “transient response”, and more specifically, as the “impulse response” of the beam, and is a typical way to look at a time-domain response. For example, if you wanted to measure how well a car suspension you designed works, you would measure its impulse response, which in physical terms is how the car bounces up and down in response to driving over a bump.

Calibrating the accelerometer

To accurately analyze the beam motion, you need to calibrate the accelerometer to make sense of its output. “Calibrating” a sensor refers to the process of measuring what output from the sensor corresponds to what level of the measured variable. For the accelerometer, you need to know how the output voltage and acceleration correspond. You can use gravity as your known acceleration, and measure the voltage output corresponding to gravity. In this case, we want the reading from the accelerometer to measure 1 g (9.8m/s^2) when it is laying on its side, and 0g when the vibration unit standing upright. See figure 1 for a graphical depiction.

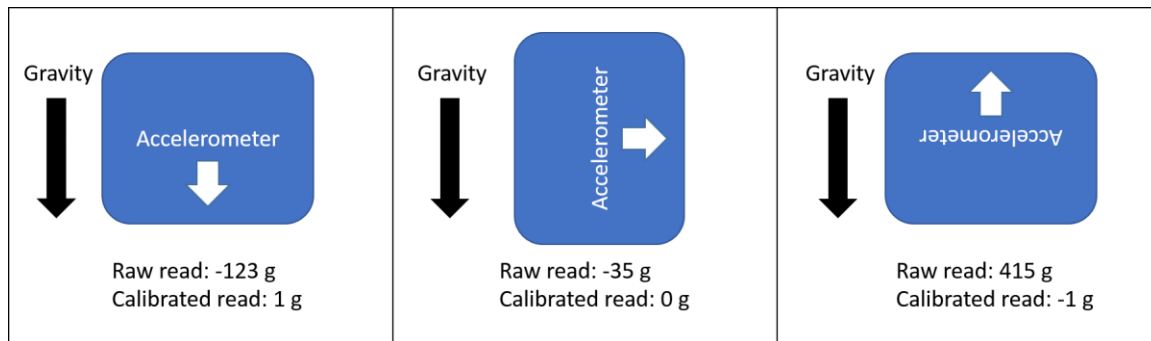


Figure 1: Example of calibrated and uncalibrated reading of accelerometer as a result of gravity and orientation.

First, make sure your accelerometer is receiving 5v/ground from the Arduino and the z terminal is connected to analog 4 (A4) on the Arduino.

Examine the Arduino code and find where the code uses the constants `accelRange` (which is set to 51) and `accelMean` (which is set to 263). The steps used to find these values were:

1. Set `accelRange = 1` and `accelMean = 0` and recompile. This now prints (to the serial monitor) the raw analog/digital counts from the accelerometer.
2. Record the accelerometer counts at rest (i.e. execute the same program but don't twang the beam).
3. Rotate the entire apparatus on its side so that the accelerometer reads the acceleration of gravity (1 g). Report the accelerometer output corresponding to 1 g.

To calibrate your accelerometer, you can use an equation of the form:

$$y = \frac{x - accelMean}{accelRange}$$

Where x is the raw input from the accelerometer and y is the value you read from the Serial Monitor.

Assuming that the counts are linearly related to the sensed acceleration, how should you change the values of accelRange and accelMean so that the accelerometer output is displayed in units of “g’s”? (i.e. it reads 1 g when held on its side)? Answer: Fit a line to the two data points you just took, where (x1,y1) = (acc1,counts1) and (x2,y2) = (acc2,counts2). The equation of the line will be of the form $y = m(x - x_{offset})$, or $acc = (counts - accelMean)/accelRange$. Go ahead and set the constants back to their old values or you can use their new values if you feel they are more accurate, and then recompile the code.

Using Acceleration to Estimate Beam Position

You mounted an accelerometer on the beam near the center of mass of the beam. For all the subsequent analysis, assume that the length of the beam is from the clamped end to the center of the accelerometer. You can then assume that the acceleration of the beam is a good measure of its position. The reason for this assumption is that the equation of motion of the unforced system has the form:

$$m\ddot{x} + c\dot{x} + kx = 0$$

Notice that if the damping of the beam is small (i.e. $c \sim 0$), then $\ddot{x} = (-k/m)x$.

For this portion of the lab we will look at the impulse response of the beam by applying an impulse to it and measuring its response with the accelerometer. The steps are:

1. Download and run the Arduino program for measuring the impulse response from the course web site.
2. Open the Serial Monitor in the Arduino IDE in order to output the accelerometer's readings at 100Hz.
3. Apply an impulse input to the beam. You just need to twang the beam with your hand to make it vibrate (make sure to hold the base still).
4. Plot the impulse response of the beam by plotting the accelerometer reading versus sample number. Note that you can convert sample number to time by multiplying sample number by the sampling period. *Save this data as you will need it for your write-up.*

Measuring the natural frequency and damping ratio. Two key parameters used to describe a vibrating system are the “damped natural frequency” and the “damping ratio”.

The damped natural frequency is the frequency that the beam oscillates after an impulse input. To measure the natural frequency you need to measure the period between oscillations.

The “damping ratio” of the beam determines how quickly the beam oscillations die out. You can estimate the damping ratio using the logarithmic decrement method:

1. After twanging the beam (impulse input), measure the initial acceleration amplitude (A_0) at the first acceleration peak. After a number of cycles N (counting the first

- peak as peak “0”), measure the acceleration amplitude again (A_N). Hint: Using the first 10 peaks is usually sufficient.
2. Calculate the damping ratio using the following equation for the logarithmic decrement method.

$$\delta = \frac{1}{N} \ln \left(\frac{A_0}{A_N} \right)$$

$$\zeta = \frac{\frac{\delta}{2\pi}}{\sqrt{1 + \left(\frac{\delta}{2\pi} \right)^2}}$$

Repeat this process five times and calculate the mean and standard deviation of your estimate of the damped natural frequency and damping ratio. Save these values for your write-up.

Practical Exam 1

Show the TA all five measurements of your estimated damped natural frequency and damping ratio. What was your final value for these parameters? Inform the TA of the proper value for accelRange and accelMean.

Part II: Frequency Domain Analysis (Forced Response)

As we have seen in class, designers often think about mechanical systems in terms of their frequency response. In this part of the lab, you will determine how the beam responds when you apply sinusoidal forces to it at different frequencies; i.e. you will measure its frequency response.

Resonance: As you observe the frequency response, a key phenomenon that you will observe is resonance. Resonance can be very destructive to a designed system, or in some cases, a designed system might want to take advantage of resonance (such as a swing set).

For this part of the lab, we will use a motor with an off-balance load to apply the sinusoidal forces to the beam. You will use the Arduino to drive the motor at a desired velocity and use the accelerometer to measure the movement of the beam (the accelerometer is the red device on the far right of Figure 3). The pot is used as an input device to control the velocity of the motor. That is, you will read in the pot voltage values in real time into the Arduino. The Arduino then sets the desired velocity of the proportional velocity controller for the motor based on these voltages. This will allow you to continuously vary the speed of the motor to try to find the resonant frequency of the beam. Again, make sure to hold the fixture base securely as it can begin to move around on the table when it is resonating.

To study the forced response:

1. Wire your motor and accelerometer as shown in Figure 3.
2. Download, compile, and load the Arduino program for measuring the frequency response of the beam.
3. Firmly secure the base of the system using a c-clamp.
4. Try to find the resonant frequency of the beam by getting it to resonate. Try not to let the system shake too badly, i.e. do not let the system stay in resonance too long.
5. Record the input frequency and peak accelerometer amplitude by printing this information to the serial monitor.

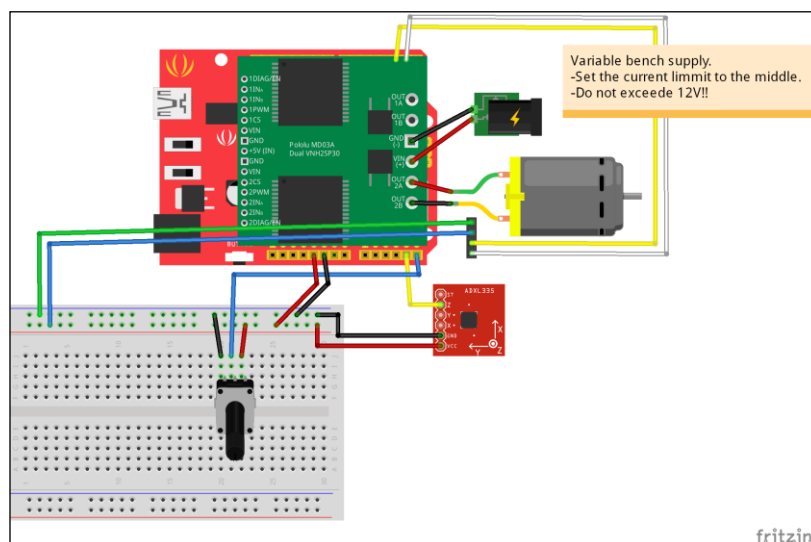


Figure 1. Wiring diagram for vibrating beam fixture.

Practical Exam 2:

Demonstrate to the TA that you can drive your beam into resonance. Explain the frequency response of your system. Draw a sketch of the magnitude Bode plot (no numbers required).

Before you leave, take data during a range of forced response frequencies. Record data at two frequencies below and two frequencies above the resonant frequency as well as the resonant frequency itself. You will use this data in the write-up.

Part III: Vibration of a system with two masses

Many mechanical systems have more than one vibrating part, and in this case the parts interact to produce different vibration patterns. Understanding these patterns is essential for predicting the behavior of your designed system. Sometimes a designer will even intentionally introduce a second vibrating part to try to cancel the vibrations produced in normal course of operation of a machine. This is a common “vibration isolation” strategy for equipment such as washing machines.

For this part of the lab, we will consider a system with two vibrating masses. You will try to identify three distinct movement patterns of the beam – two vibration modes, and a vibration cancellation mode. Mathematically, the vibration isolation frequency corresponds to a zero in the transfer function.

To identify these three movement patterns:

1. Connect a rubber band between the two beams.
2. Adjust the input frequency of the motor (i.e. its rotational speed) to see how many modes of vibration you can observe.
 - a. At a low frequency, you should be able to see the beams resonating in phase.
 - b. As you increase the frequency slightly, you should be able to get the beam with the motor attached to it to stay relatively still while the other beam oscillates a lot. This is the vibration isolation frequency. Be patient in finding this mode; you may need to wait a few seconds after each adjustment of speed to let transients die out.
 - c. Finally, as you increase the frequency more, you should be able to get the beams to resonate 180 degrees out of phase. We will derive the mathematical equations that describe these three behaviors in a later lecture.

Practical Exam 3

Demonstrate to the TA one of the resonant modes and the vibration isolation phenomenon.

Write-Up

1. Derive the transfer function for the mass-spring-damper system we used in lab. Remember that the equation of motion is of the form:

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

2. Report the mean and standard deviation of the estimated damped natural frequency and the damping ratio over at least five impulse response measurements.
3. Using MATLAB or Excel, plot the magnitude bode plot from the acceleration data in Part II. Assume values well below the resonant frequency are not amplified (gain=1), thus their decibel value is equal to 0 ($20\log(1)=0$). Estimate the interpolated lines between data points.
4. Estimate the resonant frequency of the beam given your measurements of the damped natural frequency and the damping ratio. How close is this estimate to the resonant frequency you measured experimentally?

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$$

$$\zeta \leq 0.707$$