

# 1 INTRODUCTION

## 1.1 Notations

$\forall$	For all
$\perp$	Orthogonal
$\neq$	Not Equal
$\subset$	Is a proper subset of
$\subseteq$	Subset (may be the same)
$\in$	Belongs to
$\bar{x}$	complex conjugate of $x$
$\exists$	There exists
$\exists!$	There exists a unique
$\ni$	Such that
<i>s.t.</i>	Such that
$\cup$	Union
$\cap$	Intersection
$\Rightarrow$	Implies
$\Leftrightarrow$	If and only if
iff	If and only if
$\oplus$	Direct sum of subspaces
$\perp\oplus$	Direct sum of orthogonal subspaces
$\triangleq$	Definition
$\mathcal{R}$	Real line (number)
$\mathcal{R}^+$	Positive real line (numbers)
$\mathcal{C}$	Complex scalars
$\mathcal{R}^n$	Space of n-vectors, each entry in $\mathcal{R}$
$\mathcal{C}^n$	Space of n-vectors, each entry in $\mathcal{C}$

## 1.2 Brief Introduction

This hand-out is an attempt to cover the material needed in the class. It is meant to be used as your primary source of information. Together with the references, it should give you a good start to your pursuit of research topics in the controls area. As far as overview is concerned (!) we will be following a rather strange path that connects the old fashioned SISO (single input single output) control techniques to the modern control techniques you have seen by now.

Classical control techniques were immensely powerful. To this date, it is unlikely (but not impossible!) that a new graduate student with lots of new ‘modern’ tricks can do a better job designing a control law for a SISO system (of low to moderate order), than an engineer who has a few years of experience with Bode and Nyquist plots, Bode integrals, etc. Admittedly, new techniques have given a few new wrinkles in the analysis part, but not much can be said about the synthesis part.

The problem encountered since the 60’s are: Large order systems (which make transfer functions hard to deal with), MIMO settings and the notation of explicit parameter variation (or structured uncertainties). All of these could be handled (to varying degrees of success) with modern (a.k.a., state space based) methods. This was the main focus of work in the 60’s and early 70’s.

In the late 70’s, people noticed that the ‘down side’ of using these new methods was loss or degradation of some of the classical (and critical) properties of control systems. Since then, there has been a great deal of attention paid to bridging these two approaches in an attempt to address all of the new and old concerns.

It all started in the LQR and LQG setting (later called  $H_2$  for marketing purposes) which eventually lead to the world famous  $H_\infty$  framework. More recently, the  $H_\infty$  framework has been used to develop results for a variety of more difficult problems; such as time varying problems, multi-objective problems, hybrid systems etc. The real breakthrough, though, is recent development of powerful numerical tools for finding solution to convex searches (e.g., via the LMI toolbox of MATLAB!!). This has allowed a large number of problems be solved numerically, even though closed form solutions are not apparent. In order to appreciate this progress however, a load of basic results from linear algebra and control theory is needed. We start with the basic stuff for quite a few weeks, so that we can spend the last 3 weeks or so on the new results.

Please read carefully and report any typos (for which I accept no responsibility!) to the authorities.